VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD B.E. (CBCS) III-Semester Main Examinations, December-2017

Partial Differential Equations \& Numerical Methods
Time: $\mathbf{3}$ hours
Max. Marks: 70
Note: Answer ALL questions in Part-A and any FIVE from Part-B
Part-A ( $10 \times 2=20$ Marks)

1. Express $f(x)=x$ as a Fourier series in the interval $-\pi<x<\pi$
2. Write Dirichlet's conditions.
3. Obtain the Partial differential equation by eliminating the arbitrary function $f$ from $f\left(x+y z, x^{2}+y^{2}-z^{2}\right)=0$
4. Find the complete integral of $p^{2} q^{2}(p x+q y-z)=2$
5. Use Method of separation of variables to solve $\frac{\partial u}{\partial x}=2 \frac{\partial u}{\partial t}+u$ where $u(x, 0)=6 e^{-3 x}$
6. Write one-dimensional Heat equation
7. Find a real root of the equation $x^{3}-5 x+1=0$ using Bisection method.
8. Evaluate $\Delta \tan ^{-1} x$
9. Fit a straight line $y=a+b x$ for the following data.

| $x$ | 0 | 1 | 3 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 1 | 3 | 2 | 5 | 4 |

10. If two regression lines are $3 x+2 y=26$ and $6 x+y=31$.then find the mean values and the correlation coefficient between $x$ and $y$

$$
\begin{equation*}
\text { Part-B }(5 \times 10=50 \text { Marks }) \tag{5}
\end{equation*}
$$

11. a) Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2 \pi$
b) Find the Fourier series expansion for $f(x)=\left\{\begin{array}{cc}-\pi, & -\pi<x<0 \\ x, & 0<x<\pi\end{array}\right.$

Hence show that $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\cdots--=\frac{\pi^{2}}{8}$
12. a) Solve $6 y z-6 p x y-3 q y^{2}+p q=0$ by Charpit's method.
b) Solve $\left(x+y^{2}\right) p+y q=z+x^{2}$.
13. a) A tightly stretched string of length $l$ with fixed ends is initially in equilibrium Position.

It is set vibrating by giving each point a velocity $v_{0} \sin ^{3} \frac{\pi x}{l}$. Find the displacement $y(x, t)$
b) Find the solution of Laplace equation by the Method of separation of variables.
14. a) Apply Runge-Kutta Fourth order method to find an approximate value of $y$ for $x=0.2$
insteps of 0.1. If $\frac{d y}{d x}=x+y^{2}$ given that $y(0)=1$
b) Find the cubic polynomial for the following data.

| $\mathrm{X}:$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ | 1 | 2 | 1 | 10 |

15. a) Obtain the regression line $y$ on $x$ for the following data.

| $\mathrm{X}:$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ | 2 | 5 | 3 | 8 | 7 |

b) If $\theta$ is the acute angle between the two regression lines then show that:

$$
\tan \theta=\frac{1-r^{2}}{r} \frac{\sigma_{x} \sigma_{y}}{\sigma_{x}^{2}+\sigma_{y}^{2}}
$$

Explain the significance when $r=0$
16. a) Obtain the Fourier half- range cosine series for $f(x)=x \sin x$ in the interval $0<x<\pi$
b) Find the complete integral of $\sqrt{p}+\sqrt{q}=2 x^{2}+y$
17. Answer any two of the following:
a) Use the Lagrange's interpolation formula to find the value of $y$ when $x=10$ for the following data.

| $\mathrm{X}:$ | 5 | 6 | 9 | 11 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ | 12 | 13 | 14 | 16 |

b) Derive the normal equations by the method of least squares for the straight line $y=a+b x$
c) Solve one dimensional heat equation by variables separable method.

